COHERENT CONTROL OF ULTRACOLD MATTER:
FRACTIONAL QUANTUM HALL PHYSICS AND
LARGE-AREA ATOM INTERFEROMETRY

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We describe our efforts to study the physics of the fractional quantum Hall effect using ultracold quantum gases in an optical lattice and to perform precision measurements using large-area atom interferometry.

Keywords: Fractional quantum Hall effect; coherent control; ultracold quantum gas; atom interferometry.

1. Highly Correlated States in the Fractional Quantum Hall Regime of a Rotating Bose Gas

Early after achievement of Bose-Einstein condensation of neutral atoms it was recognized that a key aspect of superfluidity in these gases was the presence of irrotational flow and quantized vorticity. It was quickly demonstrated in a series of experiments that long-lived vortices could be excited and observed in a variety of ways.1,2 Large numbers (on order of 100) vortices were soon produced in individual trapped condensates, forming the expected Abrikosov lattice of vortex cores.3 Much theoretical effort was eventually devoted to the case of extremely high vorticity, where the number of vortices is comparable to the total number of particles \(N\) in the gas, or equivalently the total angular momentum is of order \(N(N-1)\). In this fractional quantum Hall (FQH) regime, the rotation rate of the trapped gas \(\Omega\) approaches the centrifugal limit \(\Omega \rightarrow \omega\), where \(\omega\) is the harmonic trapping frequency, and the system becomes closely analogous to a two-dimensional electron gas in a magnetic field.4 In this limit, the single-particle energy
levels are organized into nearly degenerate Landau levels separated by twice the harmonic trap frequency, and the energy for the system confined to the lowest Landau level may be written as $\mathcal{H} = (\Omega - \omega)L + V$. Here, $L$ is the total angular momentum of the system, and $V$ represents repulsive contact interactions between the atoms. It was shown that many of the ground state wavefunctions discussed in the context of the FQH effect for electrons appear as ground states near the centrifugal limit of the rotating Bose gas, where the contribution to the total energy from interactions is sufficient to mix single particle states in the lowest Landau level. Similar to the situation for electrons, strong correlations are expected to occur as a result of this lifting of the large-scale single particle degeneracy. Additionally, excited states of the system have been shown to possess fractional statistical character, owing to the presence of reduced dimensionality and the presence of a gauge potential. Unfortunately, reaching the fractional quantum Hall regime with a single gas consisting of an easily probed number of atoms of order $N = 10^4$ particles requires both temperatures and precision in trap manipulation beyond the reach of previous experiments, due primarily to the inverse dependence of excitation energy on particle number. To circumvent these limitations, we describe work performed with an ensemble of rotating gases confined near the potential minima of an optical lattice potential, each of which contains a small number of particles.

To produce an optical lattice of locally rotating potentials, three laser beams of equal intensity detuned far from atomic resonance are combined with their propagation directions evenly distributed on the surface of a cone with a small apex angle of $\theta = 8^\circ$ (see Fig. 1(a)). The optical interference pattern created by these beams consists of a triangular lattice of intensity maxima, whose light shifts form a conservative trapping potential for atomic motion. Near to the minima, the potential is locally harmonic, approximately cylindrically symmetric, and may be described by $V(x, y) = -V_0 \sum_{j=1}^{3} \cos(\sqrt{3}k_e r_j)$, where $r_j = x \cos(2\pi j/3) + y \sin(2\pi j/3)$, with $x, y$ coordinates in the plane of the lattice, and $k_e = k \sin \theta$ is the reduced wavevector caused by shallow intersection.

By choosing a small intersection angle, the spacing between lattice sites may be made large, in this case $3.5\mu m$, which has the effect of reducing the tunneling rate of atoms between lattice sites in the 2D potential to a value negligible for the experiment timescale, and simultaneously making the trapping potential effectively more harmonic by reducing its vibration frequency at a fixed total depth. The three lattice beams are combined and focused to $150 \mu m$ by a single lens; each is derived from a common
1.5W, fiber-coupled beam, intensity stablized by an acousto-optic modulator and sourced from a 10W single-mode 1064nm Nd:YAG ring laser injection-locked to a stable non-planar ring oscillator (Lightwave NPRO). In order to produce a nominally cylindrically-symmetric potential near the bottom of each two-dimensional lattice site, center-of-mass vibration frequencies for loaded atoms are measured, and beam intensities adjusted to equalize these frequencies to a typical precision of 0.3%. To rotate the local potential, two electrooptic phase modulators are inserted into two of the beams forming the 2D lattice potential. By adiabatically manipulating the relative optical phase of the beams (see Fig. 1), the time-averaged potential near the lattice minima may be made to approximate a rotating anisotropic harmonic oscillator.

In order to enhance the effect of interactions between atoms loaded into this potential, an additional optical potential is applied along the axis of rotation. For this purpose two additional beams, frequency-offset from those forming the 2D potential are added, counterpropagating along the normal to the plane of the 2D lattice potential. The final potential is then a three-dimensional array of highly oblate and strongly confining ‘dot-like’ traps, approximately harmonic with radial trapping frequencies of up to 3kHz and axial frequencies up to 30kHz. Interaction strength is characterized by the dimensionless ratio of scattering length $a_s$ to oscillator length $\ell$ from confinement along the rotation direction, $\eta = a_s/\sqrt{2\pi\ell}$. For this experiment, $\eta \sim 0.07$. 

![Fig. 1. A two-dimensional triangular optical lattice is formed by the intersection of three far-detuned laser beams (a). By manipulating the relative phases of the beams, an arbitrary translation of the lattice potential in two dimensions is possible; scanning the potential rapidly (500kHz) along a given direction effectively time-averages the potential, weakening the trap curvature along the axis of translation. By slowly pivoting this axis in time (b), the time-averaged local potential approximates an anisotropic harmonic oscillator whose principal axes rotate at the rate of pivot $\Omega$ (c).]
Fig. 2. Energy levels for interacting few-atom system in a rotating trap. (a) Shows the levels for a cylindrically symmetric trap as viewed in a rotating coordinate frame for two particles interacting via a repulsive contact potential. Plot (b) shows the same spectrum after a small corotating asymmetry is added to the trap via a quadrupolar deformation ($\epsilon = 0.04$). In this case, the ground state level crossing (circled) between zero angular momentum ($L=0$) and the $L=2$ ($\frac{1}{2}$-Laughlin) state is made avoided by the perturbation to the trap, which couples levels whose angular momenta differ by $2\hbar$. In plot (c), the energy gap between the lowest energy state and first excited is plotted as a function of trap deformation strength and rotation rate.

Atoms are loaded from a $^{87}$Rb Bose-Einstein condensate of $10^5$ particles in the $|F = 2, m_F = 2\rangle$ state at a temperature $T = 30\text{nK}$ formed by evaporative cooling in a time-orbiting-potential magnetic trap with final average trap frequency $\bar{\omega} = 2\pi \times 46\text{Hz}$. After evaporation, the two-dimensional lattice potential is adiabatically increased from zero intensity to its full value of $0.5\text{W}$ per beam, loading the atoms at a peak linear density of approximately 300 atoms/$\mu\text{m}$ per tube. In order to reduce the density, the TOP trap is deformed into a quadrupole trap whose center is pulled below the position of the atoms loaded into the tubelike 2D lattice potential, and the axial confinement of atoms trapped in the two-dimensional lattice potential is adiabatically released from 42Hz to 3Hz over a time of 1s by reducing the magnetic quadrupole field, during which time the size of the cloud increases from a Thomas-Fermi diameter of 20$\mu\text{m}$ to a full-width half-max of 220$\mu\text{m}$. Following this, the axial standing wave intensity is increased to inhibit axial motion. In order to produce a well-defined mean occupancy in the full three-dimensional lattice potential, a tomographic technique is used to remove atoms far from the center of the lattice volume along the axial direction, creating a top-hat density profile. A weak magnetic field
gradient is applied, and a microwave field is applied to transfer atoms from the $|2,2\rangle$ state into $|1,1\rangle$. By slowly sweeping the microwave frequency, atoms are adiabatically transferred between internal states at the edges of the cloud. Following this, a strong field gradient is applied to completely remove atoms in the state $|1,1\rangle$. The two-dimensional lattice intensity is then slowly reduced to evaporate atoms from the center of the trap, until the desired mean occupancy is reached as inferred from absorption imaging performed transverse to the rotation axis. The lattice intensity is then returned to its full value in order to begin interrogating atoms in rotation.

In order to drive atoms from the non-rotating ground state into correlated states at nonzero angular momentum, an adiabatic pathway is followed in trap rotation rate and deformation strength. The deformation is characterized by $\epsilon = \delta \omega$, where $\omega$ and $\omega + \delta \omega$ are the minor and major axis vibration frequencies, respectively. A promising pathway can be inferred from a plot of the excitation energy from the lowest to first excited state as a function of these sweep parameters. This is shown in Figure 2, the result of a full numerical calculation for the interacting few-body system (in this case for four particles), taking into account single-particle basis functions up to a cutoff total angular momentum, here $L < 12\hbar$. It is important to note that as the particle number and angular momentum is increased, the energy of the first excited state in the centrifugal limit decreases roughly as $1/N$, suggesting the necessary ramp rate and trap precision to reach correlated ground states scale favorably only for small particle numbers. For the case of four particles, assuming an experimentally feasible interaction size of $\eta \sim 0.07$, the gap at the first ground state crossing (the four-particle Pfaffian state) is expected to be approximately $0.028\hbar \omega = 84Hz$ in an $\omega = 2\pi \times 3kHz$ trap. It is also important to note that the adiabatic transfer need not be sensitively tuned for a particular occupancy, provided in all cases one follows the ground state contour adiabatically. A representative pathway chosen for this experiment is illustrated in Figure 2c; this pathway is translated horizontally (in rotation rate) by a variable amount to provide a control parameter to probe the onset of interparticle correlations shown in Figure 3.

Once the adiabatic transfer has been completed, short range correlation in the gas is probed by applying a brief pulse of light tuned to a photoassociative transition to an electronically excited molecular state. This transfers pairs of particles found at short range (determined by the extent of the excited molecule) into short-lived molecules, whose decay is accompanied by sufficient energy to remove the constituents from the lattice trap.
The rate of observed photoassociation loss is shown in Figure 3 as a function of the final frequency of the adiabatic rotation sequence, showing a strong depression near the centrifugal limit.

![Figure 3](image)

Fig. 3. Loss of atoms following a short photoassociation pulse probes local pair correlation as a function of final rotation rate (plotted here in units of the harmonic trap frequency $\omega$) in the adiabatic pathway (a), showing strong suppression near the centrifugal limit. Average occupancy for this data set is $<N> = 5$ atoms per lattice site, as inferred from absorption imaging. Qualitative agreement can be found with full numeric evolution (b) of the four-body system, including effects of interaction, anharmonicity, and nonadiabaticity. In (b), two predicted responses are shown for differing degrees of anharmonicity, illustrating the lower-frequency downturn expected for this case. Here, $\alpha$ refers to the fractional deviation of the second single-particle vibrational splitting from the first; the parameters in part (a) correspond to $\alpha = 0.02$ at the center of the lattice volume.

A qualitative agreement can be found by comparing the measured loss to a zero-free-parameter numeric evolution (similar to that described previously by Popp, et al$^6$) of the dynamic few-body system, accounting for nonadiabaticity and anharmonicity, as shown in figure 3. Proper inclusion of the effects of anharmonicity in the local lattice potential is necessary in order to account for the detailed downturn of the photoassociation rate, as well as observed time-of-flight momentum distributions; this strongly
reduces the inferred fidelity of coupling to the strongly correlated states known from the fractional quantum Hall effect. However it is likely, given the strong modification of photoassociation rates, that these states exhibit reasonably strong correlations, as is to be expected from the strength of interactions and the relative proximity to the centrifugal limit.

2. Large-Area Atom Interferometry

Light-pulse atom interferometers\(^7\) have been used for experiments of outstanding precision, like gravimeters,\(^8\) gravity gradiometers,\(^9\) gyroscopes,\(^10\) measurements of Newton’s gravitational constant \(G\),\(^11,12\) the fine-structure constant \(\alpha\),\(^13,14\) or tests of gravitational theories.\(^15,16\) They apply the momentum \(\hbar k\) of photons to direct an atom on two (or more) paths which interfere when recombined. The sensitivity of atom interferometers increases with the phase shift between the arms. This depends linearly on the momentum splitting between the interferometer arms in gravimeters or gyroscopes — or even quadratically, like in measurements of \(\alpha\) or certain gradiometers. However, many interferometers to date are limited to a splitting of \(2\hbar k\) by the use of two-photon Raman transitions. Larger splittings of up to \(6\hbar k\) have been achieved with multiple two-photon pulses or Bragg diffraction in atomic beam setups\(^17–19\) and up to \(12\hbar k\) using Bloch oscillations.\(^20\)

We have recently made progress towards increased sensitivity in atom interferometry in several ways, that we will briefly discuss below.

2.1. Atom Interferometry with 24-Photon-Momentum-Transfer Bragg Beam Splitters

We have demonstrated the use of up to 24-photon Bragg diffraction\(^21\) as a beam splitter in light-pulse atom interferometers, the largest splitting in momentum space so far. Relative to the 2-photon processes used in the most sensitive present interferometers, these large momentum transfer (LMT) beam splitters increase the phase shift 12-fold for Mach-Zehnder (MZ-) and 144-fold for Ramsey-Bordé (RB-) geometries. We achieve a high visibility of the interference fringes (up to 52% for MZ or 36% for RB) and long pulse separation times and superior control of systematic effects that are typical of atomic fountain setups. As the atom’s internal state is not changed, important systematic effects can cancel. Figure 4 shows a gallery of interference fringes obtained in MZ and RB geometry at momentum transfers between 12-24\(\hbar k\). More details will be found in Ref. 22.
Fig. 4. A-D show MZ fringes with between 12 and 20ℏk momentum transfer; E and F are RB fringes with 12 and 24ℏk. G and H show a conjugate 20ℏk RB-pair. Throughout, T = 1 ms, T′ = 2 ms. Each data point is from a single launch (that takes 2 s), except for F, where 5-point adjacent averaging was used. The lines represent a sinewave fit.

2.2. **Noise-Immune, Recoil-Sensitive, Large-Area Atom Interferometers**

We have created a pair of simultaneous conjugate RB atom interferometers, see Fig. 5, left. Their sensitivity towards the photon recoil is similar, but the one towards inertial forces is reversed. That allows us to cancel the influence of gravity and, with simultaneous operation, noise.

Cancellation of vibrations between similar interferometers at separate locations has been demonstrated before. In some important applications, however, the interferometers must be dissimilar so that a large differential signal can be picked up. Here, we present a method to cancel vibrational noise between dissimilar interferometers, with LMT beam splitters, see Fig. 5.

The cancellation of vibrations is based on the simultaneous application of the beam splitters for the conjugate interferometers. Our experimental setup is optimized to provide the laser radiation needed with an extremely tight phase relationship; any vibrationally-induced phase shifts are thus
Fig. 5. Correlating the fringes of two interferometers creates an ellipse whose shape (eccentricity and major axis) allows to determine the relative phase.

common mode and can be taken out in an ellipse-fitting analysis of the correlation. At short pulse separation times of 1 ms, a contrast of around 25-31% is achieved at momentum transfers between $(8-20)\,\hbar k$, see Fig. 6 for examples. This should be compared to the theoretical contrast of 50%. Also, it is evident that the strong dependence of the contrast upon the momentum transfer, observed in previous LMT interferometers,\textsuperscript{22} is absent.

For $20\,\hbar k$ interferometers, about 10% contrast can be obtained at $T = 50$ ms. Without simultaneous conjugate interferometers (SCIs), this is only possible at $T = 1$ ms,\textsuperscript{22} so the use of SCIs allows us to improve the pulse separation time $T$ to 50 ms from 1 ms, without loss of contrast. This corresponds to a 2,500-fold increase in the enclosed area. At 70 ms, a contrast of 4.1% is still observable, and paves the path towards enhanced sensitivity in many cutting-edge applications. Examples include improved
measurements of the photon recoil and the fine structure constant\textsuperscript{13,14,23} and tests of the equivalence principle.\textsuperscript{16}

To further confirm the applicability of our method, we have taken 15,000 pairs of data for a 10\(\hbar k\) interferometer with a pulse separation time of 50 ms over a 12-h period, see Fig. 7. By ellipse-specific fitting, we extract the differential phase to a resolution of 6.6 ppb. This is also the resolution to which the interferometers can determine \(\hbar/M\); correspondingly, they are sensitive to the fine structure constant \(\alpha\) via
\[
\alpha^2 = \left(\frac{2R\infty}{c}\right)\left(M/m_e\right)\left(\hbar/M\right)
\]
to a resolution of 3.3 ppb.

Fig. 7. Left: 9958 Data pairs out of 15,000 that were taken during a 12-h session. Right: Histogram showing the distribution of ellipse fitting results.

2.3. Very large area atom interferometers by differential optical acceleration

The Bragg diffraction beam splitters used for LMT so far require extremely large laser power for increased momentum transfer. Even using our injection-locked 6.2 W Ti:sapph laser, which, we believe, is the strongest laser at a wavelength of 852 nm, we are limited to 20\(\hbar k\) for a reasonable contrast of the interference fringes. To increase the diffraction order, a further increase of the laser power would be required, which seems hard to achieve.

Adiabatic transfer\textsuperscript{13} or Bloch oscillations of matter waves in an accelerated optical lattice\textsuperscript{14} can be used to transfer a thousand \(\hbar k\), but this affects the common momentum of the arms, not the splitting. Here, we have developed a method that can increase the momentum transfer without being limited by the laser power.

To do so, we have first demonstrated the differential acceleration of atomic samples by Bloch oscillations in two superimposed optical lattices.
Decoupling of the samples is due to their initial momentum separation, provided by a 4\textsuperscript{th} order Bragg diffraction. A Bloch oscillation — Bragg diffraction — Bloch oscillation sequence forms a “BBB” beam splitter. Four BBB splitters make a RB atom interferometer, see Fig. 8. Two of them, running simultaneously to reject noise and systematic effects, show 15\% contrast at 24-photon-momentum splitting each, see Fig. 9.

![Space-Time diagram of simultaneous conjugate Ramsey-Bordé BBB-Interferometers](image)

**Fig. 8.** Space-Time diagram of simultaneous conjugate Ramsey-Bordé BBB-Interferometers. 1: Dual optical lattice; 2: Single Bragg beam splitter; 3: Quadruple optical lattice; 4: Dual Bragg beam splitter; a-d: outputs. The dashed lines indicate trajectories that do not interfere.

![Ellipses from simultaneous conjugate interferometers with Bragg-Bloch beam splitters](image)

**Fig. 9.** Ellipses from simultaneous conjugate interferometers with Bragg-Bloch beam splitters. The x and y axes show the normalized fluorescence of the upper and lower interferometer. (a) $\Delta p = 12\hbar k, C = 16.5\%$. (b) $\Delta p = 18\hbar k, C = 20.3\%$. (c) $\Delta p = 20\hbar k, C = 16.9\%$. (d) $\Delta p = 24\hbar k, C = 15.1\%$.

### 2.4. Towards fundamental physics measurements by atom interferometry

Taken together, the advances we just reported allow for a tremendous increase in the sensitivity of atom interferometers. We will soon apply them
for a measurement of the fine structure constant $\alpha$ at the part per billion level of accuracy. By comparison to the value derived from the electron’s anomalous gyromagnetic moment $g - 2$, this will correspond to testing the theory of quantum electrodynamics at the highest precision ever. The influence of muons and hadrons on $g - 2$ will be revealed for the first time. Moreover, this measurement would provide a limit on low-energy dark matter candidates or supersymmetric particles, and serve as a probe for the internal structure of the electron. Indeed, a measurement to 0.1 ppb would correspond to a search for physics beyond the standard model on the TeV energy scale.

References